Controlled-NOT gate design for Josephson phase qubits with tunable inductive coupling: Weyl chamber steering and area theorem

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Abstract

Superconducting qubits with tunable coupling are ideally suited for fast and accurate implementation of quantum logic. Here we present a simple approach, based on Weyl chamber steering, to CNOT gate design for inductively coupled phase qubits with tunable coupling strength g. In the presence of simultaneous rf pulses on the individual qubits that appropriately *track* the coupling strength as it is varied, we show that an infinite family of switching sequences preserving the time integral or "area" of g can be used to generate CNOT logic. We demonstrate our approach by considering time-dependencies most likely to be used in actual implementations: trapezoidal, sine, and soft quartic (also known as Landau's hat).

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Superconducting circuits containing Josephson junctions are promising candidates for scalable solid-state quantum computing architectures [1, 2, 3, 4, 5, 6, 7]. In this paper we describe a pulse switching design suitable for generation of high-fidelity CNOT logic by systems with tunable inductive coupling. Such a tunable coupling has been recently demonstrated for flux qubits [8].

The Hamiltonian (in the doubly rotating frame) for resonant phase qubits is [9]

$$H = \sum_i \frac{\Omega_i(t)}{2}\sigma_i^x + \frac{g(t)}{2}\left(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y + k\sigma_1^z\sigma_2^z\right), \tag{1}$$

where we have emphasized the fact that both the Rabi frequencies Ω_i and the qubit-qubit interaction strength g are time dependent. Here k is a constant (real) parameter of order unity that depends on the qubit flux bias. The Hamiltonian (1) neglects rapidly oscillating terms with vanishing time-averages (i.e., we use the rotating-wave approximation). When $k = 0$, the Hamiltonian (1) also describes phase qubits with tunable capacitive coupling.

Our gate construction relies on the identity

$$e^{-i\frac{\pi}{4}\left(\Lambda_1\sigma_1^x + \Lambda_2\sigma_2^x + \sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y + k\sigma_1^z\sigma_2^z\right)} = \mathrm{CNOT}_{\mathrm{Weyl}} \tag{2}$$

derived in Ref. [9], where

$$\Lambda_{1,2}(k) = \sqrt{16 - \left(\frac{k-1}{2}\right)^2} \pm \sqrt{16 - \left(\frac{k+1}{2}\right)^2}, \tag{3}$$

and

$$\mathrm{CNOT}_{\mathrm{Weyl}} \equiv e^{-i\frac{\pi}{4}\sigma_1^x\sigma_2^x} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & -i \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ -i & 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

Here $\mathrm{CNOT}_{\mathrm{Weyl}}$ is the gate in $SU(4)$ local equivalence class of canonical CNOT [10, 11, 12],

$$\mathrm{CNOT} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{5}$$

which belongs to the Weyl chamber [13]. The expression for the $\Lambda_i(k)$ given here is valid for $-7 \leq k \leq 7$; for expressions valid for larger values of $|k|$ see Ref. [9]. The CNOT can be generated (up to an overall phase factor) from $\mathrm{CNOT}_{\mathrm{Weyl}}$ by applying local $SU(2) \times SU(2)$ rotations,

$$\mathrm{CNOT} = e^{i\frac{\pi}{4}}\, e^{-i\frac{\pi}{4}\sigma_1^y}\, e^{i\frac{\pi}{4}(\sigma_1^x - \sigma_2^x)}\, \mathrm{CNOT}_{\mathrm{Weyl}}\, e^{i\frac{\pi}{4}\sigma_1^y}. \tag{6}$$

The local rotations in (6) are to be performed with $g = 0$.

An "area" theorem follows by varying the Rabi frequencies to *track* the time-dependent coupling strength,

$$\Omega_i(t) = \Lambda_i\, g(t), \tag{7}$$

with the fixed constants Λ_i given above. Then (1) becomes

$$H = g(t)\, \mathcal{H}, \tag{8}$$

with

$$\mathcal{H} \equiv \frac{\Lambda_1\sigma_1^x + \Lambda_2\sigma_2^x + \sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y + k\sigma_1^z\sigma_2^z}{2} \tag{9}$$

a fixed matrix. The Hamiltonian (8) commutes with itself at different times, so the time-evolution operator is

$$U = e^{-i\theta\mathcal{H}}, \tag{10}$$

where

$$\theta \equiv \frac{1}{\hbar}\int_0^{t_1} dt\, g(t). \tag{11}$$

Here we have assumed that the interaction is turned on at time $t = 0$ and is turned off at some later time t_1. If we choose the angle (11) to be $\theta = \pi/2$, the identity (2) shows that we can construct a CNOT gate.

Our CNOT gate implementation thus proceeds as follows:

1. First the coupling is turned off and the local rotation $R_y(-\frac{\pi}{2})_1$ on qubit 1 is performed.
2. Then g is turned on and off according to some experimentally convenient switching profile, such that
$$\int_0^{t_1} dt\, g(t) = \frac{\pi\hbar}{2}, \tag{12}$$
with $\Omega_i(t)$ tracking it in accordance with (7)
3. Simultaneous rotations $R_x(-\frac{\pi}{2})_1 \otimes R_x(\frac{\pi}{2})_2$ are applied to the qubits with $g = 0$.

4. Finally, a rotation $R_y(\frac{\pi}{2})_1$ is applied to qubit 1 with the coupling off.

We turn now to a discussion of three switching profile examples.

Trapezoidal switching — Any trapezoidal pulse [14, 15] can be broken down into three parts:

1. Tuning with
$$g(t) = \frac{gt}{\epsilon t_1}, \tag{13}$$
where $0 \leq \epsilon \leq 1/2$ characterizes the ramping fraction of the total time t_1;
2. Evolution with resonant Hamiltonian $H = g\mathcal{H}$ for $t = (1 - 2\epsilon)t_1$;
3. Detuning with
$$g(t) = g\left(1 - \frac{t}{\epsilon t_1}\right). \tag{14}$$

Then
$$\theta = \frac{gt_1}{\hbar}\,(1 - \epsilon). \tag{15}$$

We can smooth out the upper trapezoidal corners by considering inverted quadratic, quartic, and other higher order pulses with time-dependent prefactors of the form
$$g_{2n}(t) = g\left[1 - 2^{2n}\left(\frac{t}{t_1} - \frac{1}{2}\right)^{2n}\right], \quad n = 1, 2, 3, \ldots. \tag{16}$$

This leads to
$$\theta = \frac{gt_1}{\hbar}\left[1 - \frac{1}{2n+1}\right]. \tag{17}$$

Sinusoidal switching — In this case the time dependence is
$$g(t) = \frac{g}{2}\left[1 - \cos\left(\frac{2\pi t}{t_1}\right)\right], \tag{18}$$

and
$$\theta = \frac{gt_1}{2\hbar}. \tag{19}$$

Landau's hat — This pulse is in the form of the "middle part" of the famous curve used by Landau in his theory of phase transitions. It is mathematically simple, soft, and faster than sinusoidal. The time dependence is
$$g(t) = g\left(1 + (2t/t_1 - 1)^4 - 2\,(2t/t_1 - 1)^2\right), \tag{20}$$

giving
$$\theta = \frac{8gt_1}{15\hbar}. \tag{21}$$

The corresponding gate times t_1 can now be found using (11) and (12):

Switching mechanism	Pulse profile	$t_1, \times\pi\hbar/(2g)$
$\epsilon = 0.0000$ (none)	rectangular	1.0000
$\epsilon = 0.0250$ (fast)	trapezoidal	1.0256
$\epsilon = 0.2000$ (moderate)	trapezoidal	1.2500
$\epsilon = 0.5000$ (slow)	triangular	2.0000
$n = 1$	inverted quadratic	1.5000
$n = 2$	inverted quartic	1.2500
$n = 3$	inverted hexagonic	1.1667
$n = 4$	inverted octagonic	1.1250
sinusoidal	inverted cosine	2.0000
soft quartic	Landau′s hat	1.8750

(22)

In summary, we have shown how to implement a CNOT gate for phase qubits with tunable inductive coupling using a construction based on Weyl chamber steering. Two approximations have been made in our analysis, the rotating wave approximation and the neglect of leakage to higher lying (non-qubit) states.

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